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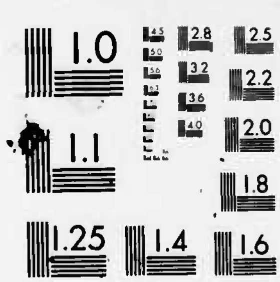

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RESISTIVE WALL INSTABILITIES IN THE MODIFIED BETATRON

Brendan B. Godfrey
Thomas P. Hughes

November 1981

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Resistive wall instabilities in modified betatrons are analyzed in several limits. The moderate frequency, negative energy, m=1 spacecharge and cyclotron waves are found to be most dangerous, potentially capable of disrupting acceleration for typical betatrons parameters. A moderate spread in electron energy can, however, stabilize these modes.

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RESISTIVE WALL INSTABILITIES IN THE MODIFIED BETATRON

Brendan B. Godfrey and Thomas P. Hughes

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ABSTRACT

Resistive wall instabilities in modified betatrons are analyzed in several limits. The moderate frequency, negative energy, m=1 spacecharge and cyclotron waves are found to be most dangerous, potentially capable of disrupting acceleration for typical betatron parameters. A moderate spread in electron energy can, however, stabilize these modes.

Spacecharge limits the circulating current in a conventional betatron to tens of amps, unless a very high initial electron energy is assumed. Adding a strong toroidal magnetic field, however, increases this limit by some two orders of magnitude.[1-3] The toroidal field also improves beam stability,[4] while a higher current has the opposite effect. This report discusses resistive wall instabilities[5] in a high current electron betatron with an applied toroidal magnetic field, referred to as a modified betatron.

Resistive-wall-driven instability occurs for both slow spacecharge and slow cyclotron beam modes, and over a wide range of frequencies.[6] Approximate analytical growth rate formulas are obtained in this report for the various branches of the resistive wall instability on a cold beam. The growth rate expressions are then evaluated for a typical set of modified betatron parameters. We find the spacecharge and cyclotron wave instabilities at frequencies comparable to the electron circulation frequency to be the most dangerous. Although slowly growing, they persist through a large fraction of the acceleration. If one can produce a spread of a few percent in electron energy, however, the modes can be stabilized. Alternatively, they can be avoided by limiting acceleration times to tens of microseconds.

The structure of the resistive wall instability dispersion relation is obtained for highly conducting cavity walls by a perturbation expansion (formally, a Rayleigh-Ritz approximation) about the beam modes in a perfectly conducting cavity. This procedure yields

$$2(\omega-\omega_0)U = P, \tag{1}$$

where ω_0 is the unperturbed wave frequency, U is the wave energy, and P is the outward Poynting flux at the cavity wall due to finite conductivity. P is evaluated easily in the long wavelength limit to be[7]

$$P = R\omega\delta B^2 e^{i\pi/4} . \qquad (2)$$

Here, R is the minor radius of the toroidal cavity, B is the wave magnetic field at the cavity wall, and δ is the skin depth,

$$\delta = \sqrt{2/\sigma\omega} . \qquad (3)$$

Frequencies and conductivities are expressed in inverse cm; the speed of light is unity. Also, a factor of 4π is absorbed into the conductivity σ.

Solving Eq. (1)-(3) for the perturbed frequency yields

$$\omega^{1/2}(\omega - \omega_0) = 2(U/\omega)^{-1}(\omega^{1/2}\delta)RB^2 e^{i\pi/4} . \qquad (4)$$

The right side of Eq. (4) is approximately independent of frequency, because $U \propto \omega$ for beam modes.[8] (This assertion is incorrect for m = 0 spacecharge waves. Nonetheless, the resulting dispersion relation has been found numerically to apply reasonably well even then.[7]) Evaluating the wave energy to complete the derivation is straightforward but tedious. Instead, we extract the needed term from the recent work of Sprangle and Vomvoridis.[9]

These authors determined the imaginary part of ω (the growth rate) to be

$$\Gamma = \frac{\Omega_z^2}{\Omega_0} \frac{n_s \gamma \delta a^2/R^3}{\Delta} . \qquad (5)$$

with

$$\Delta^2 = 1 + 2(1 - 2n_s a^2/R^2)\Omega_z^2/\Omega_\theta^2 \tag{6}$$

for moderate frequency m=1 waves. The beam radius is a. Otherwise, symbols are defined as in Ref. 9. Recasting Eq. (5) as

$$\Gamma = \frac{\omega_p^2 \delta a^2/R^3}{2\Omega_\theta \Delta} , \tag{7}$$

we see that toroidal corrections to the cylindrical drift tube dispersion relation enter only through Δ. Comparison of Eqs. (4) and (7) yields

$$\omega^{1/2}(\omega - \omega_o) = \frac{\omega_p^2 a^2/R^3}{\sigma^{1/2}\Omega_\theta \Delta} \quad f_\pm \; e^{i\pi/4} \tag{8}$$

For increased generality we have included a geometrical factor f by analogy with cylindrical drift tube results.[7]

$$f_\pm = (m^2 + \ell^2 R^2/R_o^2) \left[\frac{I_{m \pm 1}(\ell a/R_o)}{I_{m+1}(\ell R/R_o) + I_{m-1}(\ell R/R_o)}\right] \tag{9}$$

R_o is the major radius of the torus, I and K are modified Bessel functions, and ℓ amd m are the toroidal and poloidal wave numbers. With a/R and $\ell R/R_o$ both small, f is aproximately 1 for m = 1 and varies as $m^2(a/R)^{2(m-1)}$ for m > 1. The instability is weaker for m = 0 where f falls off as $(\ell a/R_o)$,^{2} and for m < 0 (wave poloidal helicity opposite that of the electrons), where f decreases as $m(a/R)^{2(m+1)}$

For slow cyclotron waves, characterized by

$$\omega \approx \ell\, \Omega_z/\gamma - \Omega_\theta/\gamma , \tag{10}$$

the peak growth rate from Eq. (8) occurs at $\omega_0 \approx 0$.

$$\omega = \sigma^{-1/3} \left[\frac{\omega_p{}^2 a^2/R^3}{\Omega_\theta \Delta} \quad f_\pm \right]^{2/3} e^{i\pi/6} . \tag{11}$$

The constraint $\omega_0 \approx 0$ implies $\ell \approx \Omega_\theta/\Omega_z$. During acceleration, the vertical magnetic field Ω_z (and the beam energy γ, which is proportional to Ω_z) increases adiabatically. Hence $\omega_0 \approx 0$ is satisfied for only a limited time period,

$$\Delta\omega \approx \frac{\Omega_\theta}{\gamma} \quad \frac{d\gamma}{dt} \; \Delta t \tag{12}$$

Equating the instability band width $\Delta\omega$ to the growth rate Γ, replacing Ω_θ by $\ell\Omega_z$, and solving for Δt, we obtain the total growth occurring as the instability passes through resonance

$$\Gamma \; \Delta t \sim \frac{\Gamma^2}{\ell(\Omega_z/\gamma)} \quad \frac{\gamma}{d\gamma/dt} \quad . \tag{13}$$

Eq. (13) is maximized late in the acceleration cycle, when γ is large and the resonant ℓ value is small.

After passing through $\omega_0 \approx 0$, the cyclotron resistive wall instability for fixed ℓ does not, of course, cease but instead transitions smoothly to a lower growth rate regime.

$$\omega = \omega_0 + \sigma^{-1/2} \omega_0{}^{-1/2} \left[\frac{\omega_p{}^2 a^2/R^3}{\Omega_\theta \Delta} \quad f_\pm \right] \quad e^{i\pi/4} . \tag{14}$$

Setting m = 1 recovers Eq. (5). Although the growth rate of Eq. (14) is somewhat smaller than that of Eq. (11), the corresponding total growth of the former,

$$(\Gamma\Delta t)_{eff} \sim (2\sigma)^{-1/2} \left[\frac{\omega_p{}^2 a^2/R^3}{\Omega_\theta} \quad f_\pm \right] \int \frac{dt}{(\omega_0 \Delta)^{1/2}} , \tag{15}$$

may exceed Eq. (13). The integration in Eq. (15) extends from the time at which ω_0 is greater than a few times the ω in Eq. (11) until the end of the acceleration.

Parallel instabilities occur for slow spacecharge waves, characterized by

$$\omega_0 \approx \ell\Omega_z/\gamma - m\omega_B , \qquad (16)$$

where ω_B is the beam poloidal rotation frequency, a small quantity.[9] For growth to occur at the rate in Eq. (11), we must have $m \approx \ell\Omega_z/\gamma\omega_B$. Even for ℓ as small as one (there is no $\ell = m = 0$ spacecharge wave), m must be quite large. Consequently, $f_\pm$ always is small near $\omega_0 = 0$ for spacecharge modes, and resistive wall growth is in this case negligible. On the other hand, for modest positive values of ω_0 instability of slow spacecharge waves is described by Eq. (14), with growth rates comparable to those of slow cyclotron waves.

Breizman and Ryutov have shown the existence of another, high frequency branch of the resistive wall instability for spacecharge modes.[10,11] To second order in R/R_0, the growth rate for this instability is identical to that in a straight tube. Peak growth for the m = 0 mode occurs at $\ell = \gamma R_0/R$, with γ the normalized electron energy.

$$\Gamma = \frac{1}{R(1 + 2\ell n(R/a))}\left(\frac{\nu}{2\sigma R}\right)^{1/2} \qquad (17)$$

Note that this expression is independent of the magnetic guide-field strength. At long wavelengths the growth rate falls off as $\ell^{1/2}$, until it merges with the lower frequency branch. The growth rate decreases more gradually at shorter wavelengths. Interestingly, the instability persists for $m \neq 0$, although with reduced growth rates.[7]

Numerical studies with the laminar beam stability code GRADR[12,13] confirm the absence of a corresponding high frequency cyclotron wave resistive wall instability.[7]

We illustrate the relative importance of the various forms of the resistive wall instability for the typical modified betatron parameters listed in Table 1. A stainless steel tube wall, with normalized conductivity $\sigma = 5.24 \cdot 10^6$ cm^{-1}, is assumed. (The conductivity of copper is about forty times larger.) Inserting these values into Eq (11), we find $\Gamma = 4.0 \cdot 10^{-5}$ cm^{-1} for m = 1 cyclotron waves at $\omega_0 \approx 0$. The corresponding skin depth is $\delta = 0.07$ cm; the cavity wall must be at least this thick for Eq. (11) to be valid. The growth rate value just given assumes $\Delta = 1$. The increase of Δ near the end of the acceleration period reduces Γ by less than 25%. Equation (13) is maximized by $\gamma \sim 70$, where the ℓ value for the $\omega_0 \approx 0$ resonance drops to one. Total growth is $\Gamma\Delta t = 3.6$ in this case. After passing through the $\omega_0 \approx 0$ resonance, the $\ell = m = 1$ slow cyclotron mode continues to grow, at the rate in Eq. (14). Evaluating this expression for the typical frequency $\omega_0 \approx \Omega_z/\gamma$ yields $\Gamma = 5.1 \cdot 10^{-6}$ cm^{-1}. Growth occurs over about 1/3 the acceleration period, or $\Gamma\Delta t = 55$ when the $\omega_0 \approx 0$ contribution is included.

As already discussed, growth of slow spacecharge waves near $\omega_0 = 0$ is negligible. At higher frequencies. the growth rate of the $\ell = m = 1$ spacecharge wave is comparable to that of the corresponding cyclotron mode. Since growth occurs throughout the acceleration period, $\Gamma\Delta t = 150$. At still higher frequencies m = 0 spacecharge waves grow at a maximum rate of $\Gamma = 1.3 \cdot 10^{-6}$, or about 25% of the intermediate frequency growth rate. It is important to bear in mind that these growth estimates probably are uncertain by a factor of two, due to approximations made.

TABLE 1

Typical Modified Betatron Parameters Used in Evaluating Resistive Wall Instability Growth Rates.

Toroidal Magnetic Field	B_θ	2.5 kg
Vertical Magnetic Field (Initial)	B_z	115 g
Toroid Major Radius	R_o	100 cm
Toroid Minor Radius	R	10 cm
Beam Radius	a	1 cm
Beam Current	ν	0.59
Beam Energy	γ	7-100
Acceleration Time	τ	$3 \cdot 10^7$ cm

The preceding analysis ignores thermal spread in the beam electron velocities. As a result of the toroidal geometry, a spread in electron toroidal velocity can smear out the transverse resonances associated with the resistive instabilities. In a companion paper we find that the required spreads in toroidal and transverse velocities are achievable from a few percent spread in initial electron kinetic energy.[14] A similar conclusion was drawn in Ref. 9, based on a Lorentzian distribution of energies. Of course, this damping of spacecharge and cyclotron wave instabilities by a spread in electron velocities is ineffective for low frequency modes. As we have already seen, however, these latter modes are not a serious problem.

In summary, we find that the $m = 1$, low ℓ spacecharge and cyclotron wave instabilities are the most dangerous of the various resistive wall phenomena identified for modified betatrons. Amplification factors of $e^{150} \sim 10^{65}$ and $e^{55} \sim 10^{24}$, respectively, are predicted for the parameters in Table 1. Successful operation of a modified betatron requires cutting the growth exponent $\Gamma\Delta t$ to about unity. (Computer simulations suggest that initial perturbations may be quite large.[15]) Employing more highly conducting cavity walls provides a factor of six. Reducing the acceleration time by an order of magnitude would then effectively eliminate these instabilities. Alternatively, an initial spread of a few percent in the electron energy should be sufficient to damp out the modes.

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